tions along the ray PM orthogonal to Γ at the point P, for the values of time corresponding to $\varkappa\tau / a^2 = 0.04$ (curves 1) and $\varkappa\tau / a^2 = 0.09$ (curves 2), and the values of t_p equal to 0(a), $\pi / 4$(b) and $\pi / 2$ (c).

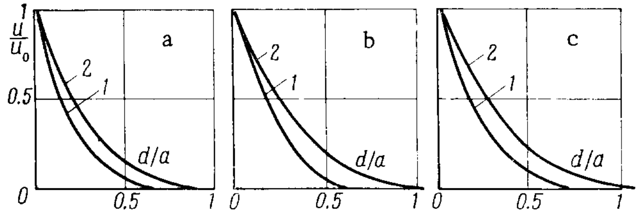

Fig. 1

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DETERMINATION OF THE FREQUENCY OF THE APPROXIMATE SOLUTION OF HILL'S EQUATION

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In the theory of motion of charged particles through periodic focussing accelerators the Hill's equation is often solved using a widely accepted method of "smooth approximation". By this method the solution is represented in the form of a "slow" harmonic function with a "rapidly" oscillating amplitude. Below we derive a formula for the frequency of the slow component of such a solution, expressed in terms of the Fourier harmonics of the equation coefficient. Such a formula may find use in practical computations.

In the smooth approximation [1] which converges to the first approximation of the method of averaging [2] the solution of the Hill equation

$$x^{\cdot\cdot} + q(t)x = 0, \qquad q(t + T) \equiv q(t) \qquad (T > 0) \tag{1}$$

is sought in the form $x(t) = [1 + r(t)]X(t)$, where $X(t)$ represents a slow (compared with $r(t)$) function and $r(t)$ is uniquely determined by the conditions

$$r^{\cdot\cdot} = -q(t) + \langle q\rangle \equiv -\Delta q(t) \tag{2}$$

$$\langle r^{\cdot}\rangle = 0, \quad \langle r\rangle = 0 \tag{3}$$

Here the symbol < > denotes averaging over the period of the coefficient $q(t)$. The function $X(t)$ approximately satisfies the equation

$$X^{\cdot\cdot} + \omega^2 X = 0 \qquad (\omega^2 = \langle q\rangle + \langle r^{\cdot 2}\rangle) \tag{4}$$

The smooth approximation is applicable when $\omega \ll 2\pi / T$. Let us express ω^2 by the coefficients of the Fourier expansion of $q(t)$

$$q(t) = \langle q\rangle + \sum_{n=1}^{\infty}\left(a_n \cos\frac{2\pi nt}{T} + b_n \sin\frac{2\pi nt}{T}\right) \tag{5}$$

We assume that the function $q(t)$ is bounded, piecewise monotonous and has a finite number of discontinuities within a single period. Under these conditions the Fourier series (5) converges and can be integrated term by term. Equation (2), with the expansion (5) and the first equation of (3) taken into account, yields

$$r^{\cdot} = \sum_{n=1}^{\infty}\left(A_n \cos\frac{2\pi nt}{T} + B_n \sin\frac{2\pi nt}{T}\right) \tag{6}$$

$$A_n = \frac{b_n}{2\pi n / T}, \qquad B_n = -\frac{a_n}{2\pi n / T} \tag{7}$$

Function $r^{\cdot}(t)$ is periodic and continuous, therefore it satisfies the Parseval equation

$$\langle r^{\cdot 2}\rangle - \langle r^{\cdot}\rangle^2 = \frac{1}{2}\sum_{n=1}^{\infty}(A_n^2 + B_n^2) \tag{8}$$

Taking into account the fact that $\langle r^{\cdot}\rangle = 0$, we obtain from (7) and (8)

$$\omega^2 = \langle q\rangle + \frac{T^2}{8\pi^2}\sum_{n=1}^{\infty}\frac{a_n^2 + b_n^2}{n^2} \tag{9}$$

Computation of ω according to the formula (9) may be more convenient than direct integrating of (2) with the condition (3) taken into account, in the case when e. g. the coefficient $q(t)$ is either defined by different expressions on different segments of the period, or when it is given in tabular form. Using the formula (9) we can write the conditions of simultaneous stability of the solutions of (1) and of the equation

$$x^{\cdot\cdot} - 2q(t)\,x = 0 \tag{10}$$

in the following manner (the problem relating to the theory of accelerators):

$$\sum_{n=1}^{\infty}\frac{a_n^2 + b_n^2}{n^2} > \begin{cases} 4\pi^2 T^{-2}\langle q\rangle, & \langle q\rangle > 0 \\ 8\pi^2 T^{-2}\,|\langle q\rangle|, & \langle q\rangle < 0 \end{cases}$$

These conditions show clearly the role that the harmonics play in securing simultaneous stability of the solutions of (1) and (10).

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